

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
SECOND SEMESTER B.TECH DEGREE EXAMINATION, JULY 2018

Course Code: MA102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks

Marks

- | | | |
|----|---|-----|
| 1 | Consider the initial value problem $y'' - x^3 y' + 6xy = \sin x$, $y(0)=3, y'(0)=-1$.
Can this problem have unique solution in an interval containing zero? Explain. | (3) |
| 2 | Find any three independent solutions of the differential equation $y''' - y' = 0$. | (3) |
| 3 | Find the particular solution of the differential equation $y'' - 6y' + 9y = e^{3x}$. | (3) |
| 4 | Using a suitable transformation, convert the differential equation $(2x-3)^2 y'' - (2x-3)y' + 2y = (2x-3)^2$ into a linear differential equation with constant coefficients. | (3) |
| 5 | State the conditions for which a function $f(x)$ can be represented as a Fourier series. | (3) |
| 6 | Discuss the convergence of a Fourier series of a periodic function $f(x)$ of period 2π . | (3) |
| 7 | Find the partial differential equation representing the family of spheres whose centers lies on z-axis. | (3) |
| 8 | Find the particular solution of $(D^2 - 2DD' + 2D'^2)z = \sin(x-y)$ | (3) |
| 9 | Write any three assumptions involved in the derivation of one dimensional wave equation. | (3) |
| 10 | A string of length l fastened at both ends. The midpoint of the string is taken to a height h and then released from rest in that position. Write the boundary conditions and initial conditions of the string to find the displacement function $y(x,t)$ satisfying the one dimensional wave equation. | (3) |
| 11 | Write the fundamental postulates used in the derivation of one dimensional heat equation. | (3) |
| 12 | Define steady state condition in one dimensional heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$. | (3) |

PART B

Answer six questions, one full question from each module

Module 1

- | | | |
|----|---|-----|
| 13 | a) Discuss the existence and uniqueness of solution of the initial value problem $\frac{dy}{dx} = \frac{y}{\sqrt{x}}, y(1)=3$. | (6) |
| | b) Prove that $y_1(x)=e^x$ and $y_2(x)=e^{4x}$ form a fundamental system(basis) for the | (5) |

differential equation $y'' - 5y' + 4y = 0$. Can $5e^x - 2e^{4x}$ be a solution (do not use verification method) of the differential equation? Explain.

OR

- 14 a) Discuss the existence and uniqueness of solution of the initial value problem (6)

$$\frac{dy}{dx} = x^2 + y^2, y(0)=1 \text{ in the rectangle } |x| \leq 1, |y-1| \leq 1.$$

- b) If $y_1(x)=x$ is a solution of $x^2y'' + 2xy' - 2y = 0$, find the general solution. (5)

Module II

- 15 a) By the method of variation of parameters, solve $y'' + y = x \sin x$. (6)

- b) Solve $y'' + 5y' + 6y = e^{-2x} \sin 2x$. (5)

OR

- 16 a) Solve $x^2y'' + xy' - 9y = \log x$. (6)

- b) Solve $y'' - 2y' + 5y = x^2$. (5)

Module III

- 17 Find the Fourier cosine series representation of $f(x)=x, 0 \leq x \leq \pi$. Also find the Fourier series representation $f(x)$ if $f(x)$ is periodic function with period π . (11)

OR

- 18 Find the Fourier series of the periodic function $f(x)$ of period 4, where (11)

$$f(x) = \begin{cases} 2, & -2 < x \leq 0 \\ x, & 0 < x < 2 \end{cases} \text{ and deduce that}$$

$$(i) \ 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8} \text{ and } (ii) \ 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$$

Module IV

- 19 a) Find the particular solution of $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = y^2$. (5)

- b) Find the general solution of $(y^2 + z^2)p - xyq = -xz$. (6)

OR

- 20 a) Solve $(D^2 + 3DD' + 2D'^2)z = (2x + y)^7$. (5)

- b) Solve $4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 16 \log(x + 2y)$. (6)

Module V

- 21 a) Using method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} - u, u(x,0) = 5e^{-3x}$. (5)

- b) A tightly stretched string of length l fastened at both ends is initially in a position given by $y = kx, 0 < x < l$. If it is released from rest from this position, find the displacement $y(x,t)$ at any time t and any distance x from the end $x = 0$. (5)

OR

- 22 A string is stretched and fastened in two points 50 cm apart. Motion is started by (10)

displacing the string into the form of the curve $y = x(50 - x)$ and also by imparting a constant velocity V to every point of the string in the position at time $t = 0$. Determine the displacement function $y(x, t)$.

Module VI

- 23 A rod of length 50 cm has its ends A and B kept at 20°C and 70°C respectively (10) until steady state temperature prevail. The temperature at each end is then suddenly reduced to zero temperature and kept so. Find the resulting temperature function $u(x, t)$ taking $x = 0$ at A.

OR

- 24 A bar 10 cm long with insulated sides has its ends A and B maintained at 50°C (10) and 100°C respectively until steady state conditions prevail. The temperature at A is suddenly raised to 90°C and at the same time that at B is lowered to 60°C . Find the temperature distribution in the bar at time t .
